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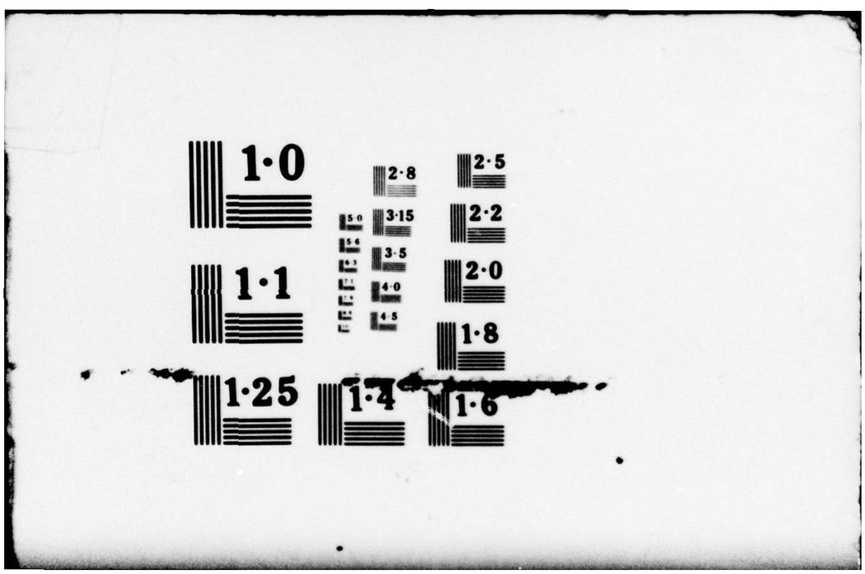
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SOLVING THE CURRENT ELEMENT PROBLEM OVER LOSSY HALF-SPACE WITHOUT SOMMERFELD INTEGRATIONS

Interim Technical Report

LEVEL II

R. Mittra
P. Parhami
Y. Rahmat-Samii

October 1978

U. S. Army Research Office
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Electromagnetics Laboratory
Department of Electrical Engineering
University of Illinois
Urbana, IL 61801

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In this paper a new approach is presented for efficient computation of the vector potentials arising in the problem of a current element radiating over a lossy half-space. The present approach departs from the conventional ones in that it works primarily with the transform domain representations rather than with the Sommerfeld integrals which are the corresponding spatial domain counterparts. The key step in the present method is to approximate the transforms using a suitable approximation which is valid for a wide range of parameters of practical interest. The approximated transforms can be

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inverted in a closed form for the horizontal component of the vector potentials (Π_x), and can be expressed in a computationally efficient form for the vertical component (Π_z). Numerical results illustrating the accuracy of the method are presented in the paper and some estimates of comparative computational times also are included.

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In this paper a new approach is presented for efficient computation of the vector potentials arising in the problem of a current element radiating over a lossy half-space. The present approach departs from the conventional ones in that it works primarily with the transform domain representations rather than with the Sommerfeld integrals which are the corresponding spatial domain counterparts. The key step in the present method is to approximate the transforms using a suitable approximation which is valid for a wide range of parameters of practical interest. The approximated transforms can be inverted in a closed form for the horizontal component of the vector potentials (Π_x), and can be expressed in a computationally efficient form for the vertical component (Π_z). Numerical results illustrating the accuracy of the method are presented in the paper and some estimates of comparative computational times also are included.

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1. Introduction

The conventional approach to analyzing antenna structures radiating in the presence of a lossy half-space involves repeated evaluation of the Sommerfeld integrals appearing in the expressions for the vector potentials [1]. Since the evaluation of these infinite integrals is an extremely time-consuming process, much attention has been focused in recent years on developing techniques for efficiently evaluating the Sommerfeld integrals without unduly sacrificing the accuracy [2-8]. However, even the latest reported techniques for evaluating the Sommerfeld integrals are 40 to 100 times slower than the Reflection Coefficient Method (RCM), which evaluates these integrals asymptotically and is valid for large kr (where k is the free space wave number, and r is the distance between the image and observation points).

In this paper, we present a new approach for rapid and accurate numerical evaluation of the vector potentials that avoids the tedious task of handling the Sommerfeld integrals. We begin with the two-dimensional Fourier transforms of the vector potentials which are conveniently expressed in simple closed forms. Next, we show that under a suitable approximation the inverse transform of the vector potentials can be performed analytically using a set of identities. The resulting space domain expressions are either expressed in a closed form or require evaluating a finite integral. These expressions are valid for a wide range of frequencies, ground parameters, and observation points.

In the following sections, the use of the above procedure is demonstrated by considering a horizontal current element over lossy ground, and several numerical examples are included to illustrate the accuracy and computational efficiency of the method. We find that the computational time is only slightly larger than the reflection coefficient method (RCM) and the accuracy of the results is good for a wide range of parameters of practical interest.

2. Transformed Vector Potentials

The fields radiated by a horizontal current element over a lossy ground (see Fig. 1) can be expressed in terms of two vector potential components Π_x and Π_z [5,8]. We define the following two-dimensional Fourier transform pair:

$$\tilde{\vec{\Pi}} = \iint_{-\infty}^{\infty} \vec{\Pi} \exp[-j(\alpha x + \beta y)]\, dx\, dy \tag{1a}$$

$$\vec{\Pi} = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \tilde{\vec{\Pi}} \exp[j(\alpha x + \beta y)]\, d\alpha\, d\beta \tag{1b}$$

where the ~ on top represents the transformed quantities. It has been shown [5,8] that, using the exp(jωt) time convention and for observation points above the lossy ground, the following expressions for the vector potentials satisfy the Maxwell's equations and the required boundary conditions:

$$\tilde{\Pi}_x = \tilde{\Pi}_x^i + \tilde{\Pi}_x^r + {}_0\tilde{\Pi}_x \tag{2a}$$

$$\tilde{\Pi}_x^i = I_0 \frac{1}{2j\,\gamma_1} \exp[-j\,\gamma_1 |z-h|] \tag{2b}$$

$$\tilde{\Pi}_x^r = I_0 \frac{-1}{2j\,\gamma_1} \exp[-j\,\gamma_1 (z+h)] \tag{2c}$$

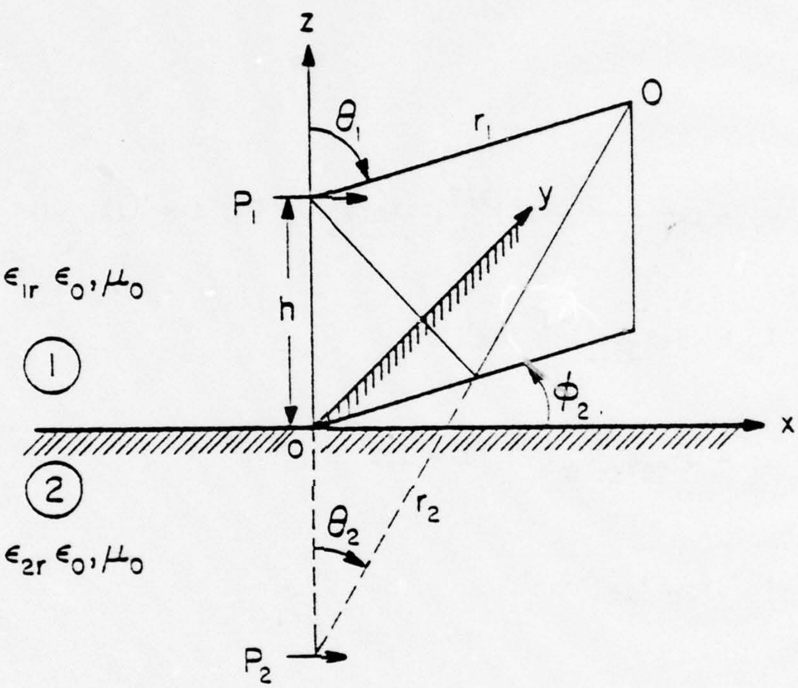

Figure 1. Geometry and the coordinate systems for the current element P_1 radiating over imperfect ground, where $\varepsilon_{1r} = 1$ and $\varepsilon_{2r} = \varepsilon_r - j\sigma/(\omega\varepsilon_0)$ has been assumed.

$$
{}_0\tilde{\Pi}_x = I_0 \frac{1}{j(\gamma_1+\gamma_2)} \exp[-j\ \gamma_1(z+h)] \tag{2d}
$$

and

$$
\tilde{\Pi}_z = I_0\ j\alpha \frac{\gamma_1-\gamma_2}{\kappa\gamma_1+\gamma_2} \exp[-j\ \gamma_1(z+h)] \tag{3}
$$

where

$$
\gamma_i = [k_i^2 - \alpha^2 - \beta^2]^{1/2}; \quad Im(\gamma_i) \leq 0 \ ; \ i = 1,2 \tag{4a}
$$

$$
I_0 = (j\omega\varepsilon_0\varepsilon_{1r})^{-1}\ Idx' \tag{4b}
$$

$$
k_i^2 = \omega^2\mu_0\varepsilon_{ir}\varepsilon_0 \quad ; \ i = 1,2 \tag{4c}
$$

$$
\kappa = \varepsilon_{2r}/\varepsilon_{1r} \ . \tag{4d}
$$

Equations (2b) and (2c) are the solutions to the problem of a current element radiating in free space and can be interpreted as the direct and the reflected contributions to the field at the observation point for a perfectly conducting ground. Their inverse transforms are expressed in the following well-known space domain forms:

$$\Pi_x^i = I_0 \exp(-j\, k_1\, r_1) \,/\, 4\pi r_1 \tag{5a}$$

$$\Pi_x^r = -I_0 \exp(-j\, k_1\, r_2) \,/\, 4\pi r_2 \tag{5b}$$

where (r_1,θ_1,ϕ_1) and (r_2,θ_2,ϕ_2) are the spherical coordinate systems erected at the source and its image point respectively. Equations (2d) and (3) are the correction terms to the perfect ground solution and are not directly amenable to inversion in closed forms. Traditionally, these two equations are transformed into space domain representations comprised of infinite integrals that are oscillatory in nature. These integral representations take many different forms and the following are examples which contain the Hankel functions:

$${}_0\Pi_x = \frac{I_0}{4\pi j} \int_{-\infty}^{\infty} \frac{\lambda}{\sqrt{k_1^2-\lambda^2} + \sqrt{\kappa k_1^2-\lambda^2}}\, H_0^{(2)}(\rho_2\lambda)\, \exp(-jz_2\sqrt{k_1^2-\lambda^2})\, d\lambda \tag{6a}$$

and

$$\Pi_z = -\frac{I_0}{4\pi k_1^2} \cos\phi_2 \int_{-\infty}^{\infty} \lambda^2\, \frac{\sqrt{k_1^2-\lambda^2} - \sqrt{\kappa k_1^2-\lambda^2}}{\kappa\sqrt{k_1^2-\lambda^2} + \sqrt{\kappa k_1^2-\lambda^2}}\, H_1^{(2)}(\rho_2\lambda)$$

$$\cdot \exp(-jz_2\sqrt{k_1^2-\lambda^2})\, d\lambda\ . \tag{6b}$$

The integrals appearing in equations (6a) and (6b), better known as the Sommerfeld integrals, are quite time-consuming to evaluate and represent a major hurdle to the task of efficient analysis of antennas located over a lossy ground. Many of the recent works in the literature [4-8] have concentrated on the problem of reducing the computational time required in the evaluation of the Sommerfeld integrals without sacrificing the accuracy beyond a tolerable level. In the following sections, an alternate approach is proposed for Fourier inverting (2d) and (3) in a more direct manner which circumvents the need for computing the Sommerfeld integrals (6a) and (6b).

3. Approximate Expressions for the Transformed Vector Potentials

An examination of the expressions for transformed vector potentials given in (2d) and (3) reveals two important and useful properties. First, both of the equations have an identical z-variation that corresponds to a space domain solution emanating from the image point of the original dipole source. Second, it is apparent that the two equations are well behaved in the Fourier transform domain and decay exponentially to zero outside the circle $\alpha^2 + \beta^2 = k_1^2$.

Starting from the transform pair (2c) and (5b), one can generate a set of useful identities by successively applying the operator $\partial/\partial z$ to the transform pair. We have, for instance

$$\tilde{Q} = \gamma_1^{n-1} \exp[-j \gamma_1 (z+h)] \tag{7a}$$

$$Q = 2(j)^{n+1} \frac{\partial^n}{\partial z^n} g \quad ; \quad n = 0,1,2,... \tag{7b}$$

where g is the free-space Green's function

$$g(x,y,z) = \exp[-j\, k_1\, r_2] \,/\, 4\pi r_2 \quad ; \; r_2 = \sqrt{x^2 + y^2 + (z+h)^2} \; . \tag{8}$$

We also note that the successive partial derivatives of g can be obtained explicitly; hence, (7b) is expressible in a closed form. We will now attempt to approximate the transformed vector potentials ${}_0\tilde{\Pi}_x$ and $\tilde{\Pi}_z$, given in (2d) and (3), such that the inverse transform operation is performed via (7a-b) without an undue sacrifice in the accuracy. To this end, we introduce the only approximation needed to accomplish this goal, by letting

$$\gamma_2 \simeq k_1 \sqrt{\kappa} \tag{9}$$

Kuo and Mei [7, Eq. 8] have employed this approximation to manipulate the Sommerfeld integrals (6) and have found it to be accurate inside the $\alpha^2 + \beta^2 = k_1^2$ circle for most practical parameter ranges of interest. In addition, outside the circle of visible range, the decaying exponential overcomes most of the errors introduced by the substitution of (9) into (2d) and (3).

3a. Approximation for Π_x

The transform ${}_0\tilde{\Pi}_x$, given in (2d), can be written in the following form

$${}_0\tilde{\Pi}_x = I_0 \frac{1}{jk_1^2\,(1-\kappa)} (\gamma_1 - \gamma_2) \exp[-j\, \gamma_1\, (z+h)] \tag{10}$$

Introducing the approximation in (9) and using the identities in (7), one can express the space-domain expression for ${}_0\Pi_x$ in a closed form

$$ {}_0\Pi_x = I_0 \frac{2\sqrt{\kappa}}{jk_1 (1-\kappa)} \frac{\partial}{\partial z} g - I_0 \frac{2}{k_1^2 (1-\kappa)} \frac{\partial^2}{\partial z^2} g \quad . \tag{11} $$

Before closing this subsection, it may be worthwhile to point out one of the key differences between the Kuo and Mei scheme [7] and the present method, both of which employ identical approximations. In contrast to the method in [7], the present approach requires no numerical integration and generates the ${}_0\Pi_x$ solution entirely in a closed form.

3b. Approximation for Π_z

The expression for Π_z, given in (3), can be rearranged into the following form

$$ \tilde{\Pi}_z = I_0 \frac{j\alpha}{k_1^2 \kappa} \left[1-(\kappa+1) \frac{\gamma_2}{\kappa\gamma_1 + \gamma_2}\right] \exp[-j \gamma_1 (z+h)] \ . \tag{12} $$

Again introducing the approximation expressed in (9), one can further simplify (12) into

$$ \tilde{\Pi}_z = I_0 \frac{j\alpha}{k_1^2 \kappa} \exp[-j \gamma_1 (z+h)] - I_0 \frac{c (\kappa+1)}{k_1^2 \kappa} \tilde{P} \tag{13} $$

where

$$ \tilde{P} = \frac{j\alpha}{\gamma_1 + c} \exp[-j \gamma_1 (z+h)] \tag{14a} $$

$$ c = k_1/\sqrt{\kappa} \quad . \tag{14b} $$

The space-domain expression for Π_z can now be expressed in terms of P, the inverse transform of $\tilde{P}$, given in (14a) as

$$\Pi_z = I_0 \frac{-2}{k_1^2 \kappa} \frac{\partial^2}{\partial x \, \partial z} g - I_0 \frac{c(\kappa+1)}{k_1^2 \kappa} P \quad . \tag{15}$$

Using the identities in (7), it can be shown that P satisfies the following first-order linear inhomogeneous differential equation

$$\frac{\partial P}{\partial z} - jcP = 2j \frac{\partial^2 g}{\partial x \, \partial z} \quad . \tag{16}$$

The boundary condition required for the above differential equation can be imposed by evaluating the asymptotic solution to the vector potential expression in (15), derived in [5], at an observation point which is sufficiently high above ground. The asymptotic solution is given by

$$\Pi_{za} \sim 2 \, I_0 \cos \phi_2 \sin \theta_2 \cos \theta_2 \frac{\cos \theta_2 - \sqrt{\kappa - \sin^2 \theta_2}}{\kappa \cos \theta_2 + \sqrt{\kappa - \sin^2 \theta_2}} \cdot \exp(-jk_1 r_2)/4\pi r_2 \tag{17a}$$

Using (17a) in (15), we arrive at

$$P(z') = \frac{-1}{c(\kappa+1)} \left\{ \frac{k_1^2 \kappa}{I_0} \Pi_{za}(z') + 2 \frac{\partial^2}{\partial x \, \partial z} g(z') \right\} \tag{17b}$$

where z' is a suitable height at which the RCM approximation in (17a) is valid. We point out that for the sake of simplicity, the x and y dependences of the P, Π_{za}, and g have been suppressed from their arguments. The solution for P can be obtained by integrating (16) and one can arrive at the following form which is convenient for numerical computation:

$(z+h)/\lambda$	$(z'+h)/\lambda$	Π_z (present method) $\times 10^3$
.42	.50	1.82 - j.953
.42	.75	1.76 - j1.39
.42	1.00	1.57 - j1.31
.42	1.25	1.64 - j1.22
.42	1.50	1.68 - j1.29
.42	1.75	1.63 - j1.30
.42	2.00	1.62 - j1.25

At $(z+h)/\lambda = .42$ Exact $\Pi_z \times 10^3 =$ 1.58 - j1.29

At $(z+h)/\lambda = .42$ RCM $\Pi_z \times 10^3 =$ 1.64 - j.773

TABLE 1. Demonstration of the stability of the present technique for evaluating Π_z, as a function of z'. For this example, $\varepsilon_r = 10$, $\sigma = .01$ mhos/m, frequency = 18 MHz, $r_2/\lambda = .6$, and $\theta_2 = 45°$.

Freq. (MHZ)	r_2/λ	Exact Sommerfeld Integration × 10^4	Asymptotic Evaluation (RCM) × 10^4	Present Method × 10^4
3.	.1	2.49 - j2.74	1.98 + j.289	2.49 - j2.74
6.	.2	1.48 - j3.26	2.49 - j1.33	1.48 - j3.26
9.	.3	-.111 - j3.88	1.49 - j3.11	-.109 - j3.88
12.	.4	-2.25 - j3.61	-.742 - j3.90	-2.24 - j3.61
15.	.5	-4.17 - j2.01	-3.24 - j3.02	-4.17 - j2.01
18.	.6	-4.94 + j.675	-4.81 - j.561	-4.94 + j.674
21.	.7	-3.96 + j3.57	-4.53 + j2.58	-3.96 + j3.57
24.	.8	-1.29 + j5.51	-2.27 + j5.09	-1.29 + j5.51
27.	.9	2.22 + j5.54	1.24 + j5.77	2.22 + j5.54
30.	1.0	5.26 + j3.39	4.64 + j4.12	5.26 + j3.39

TABLE 2. ${}_0\Pi_x$ evaluation by three techniques for $\varepsilon_r = 40$, $\sigma = 1.$ mhos/m, $r_2 = 10$ m, $\theta_2 = 10°$, and $\phi_2 = 0$.

Freq. (MHZ)	r_2/λ	Exact Sommerfeld Integration × 10^5	Asymptotic Evaluation (RCM) × 10^5	Present Method × 10^5
3.	.1	-4.38 + j4.73	-3.47 - j.473	-4.39 + j4.72
6.	.2	-2.59 + j5.70	-4.31 + j2.37	-2.60 + j5.70
9.	.3	.247 + j6.77	-2.49 + j5.44	.225 + j6.79
12.	.4	4.00 + j6.24	1.41 + j6.73	3.96 + j6.29
15.	.5	7.31 + j3.37	5.71 + j5.09	7.25 + j3.46
18.	.6	8.55 - j1.35	8.31 + j.777	8.46 - j1.22
21.	.7	6.71 - j6.35	7.69 - j4.63	6.58 - j6.15
24.	.8	1.98 - j9.57	3.67 - j8.83	1.82 - j9.30
27.	.9	-4.10 - j9.44	-2.40 - j9.82	-4.31 - j9.08
30.	1.0	-9.23 - j5.57	-8.15 - j6.81	-9.49 - j5.12

TABLE 3. Π_z evaluation by three techniques for $\varepsilon_r = 40$, $\sigma = 1.$ mhos/m, $r_2 = 10$ m, $\theta_2 = 10°$, and $\phi_2 = 0$.

Freq. (MHZ)	r_2/λ	Exact Sommerfeld Integration $\times 10^4$	Asymptotic Evaluation (RCM) $\times 10^4$	Present Method $\times 10^4$
3.	.1	21.5 – j19.8	18.3 + j.0076	24.4 – j17.3
6.	.2	9.98 – j27.2	18.3 – j16.1	11.6 – j26.6
9.	.3	-6.48 – j29.8	4.11 – j27.7	-5.27 – j29.9
12.	.4	-23.8 – j21.8	-15.7 – j26.2	-22.5 – j22.4
15.	.5	-32.9 – j4.22	-30.3 – j11.1	-32.5 – j5.09
18.	.6	-29.9 + j16.5	-31.8 + j10.4	-30.1 + j15.6
21.	.7	-14.6 + j31.6	-19.2 + j28.5	-15.2 + j31.1
24.	.8	7.10 + j34.6	2.15 + j34.9	6.33 + j34.6
27.	.9	26.6 + j24.0	23.2 + j26.9	26.0 + j24.3
30.	1.0	35.9 + j3.54	35.1 + j7.50	35.7 + j4.18

TABLE 4. ${}_0\Pi_x$ evaluation by three techniques for $\epsilon_r = 10$, $\sigma = .01$ mhos/m, $r_2 = 10$ m, $\theta_2 = 10°$, and $\phi_2 = 0°$.

Freq. (MHZ)	r_2/λ	Exact Sommerfeld Integration $\times 10^5$	Asymptotic Evaluation (RCM) $\times 10^5$	Present Method $\times 10^5$
3.	.1	-34.5 + j28.7	-29.0 + j2.68	-45.0 + j19.6
6.	.2	-14.1 + j43.1	-24.5 + j27.5	-20.1 + j41.6
9.	.3	13.9 + 44.4	-.462 + j40.8	9.95 + j45.6
12.	.4	37.9 + j27.8	27.1 + j33.3	36.2 + j30.6
15.	.5	47.2 - j.194	43.1 + j9.34	47.6 + j2.87
18.	.6	37.9 - j28.1	40.3 - j19.5	40.0 - j25.7
21.	.7	13.8 - j44.9	20.3 - j40.4	16.9 - j43.7
24.	.8	-15.3 - j44.3	-8.29 - j44.6	-12.2 - j44.2
27.	.9	-38.3 - j26.7	-33.5 - j30.8	-36.0 - j27.3
30.	1.0	-46.6 + j.971	-44.5 - j4.61	-45.2 + j.42

TABLE 5. Π_z evaluation by three techniques for $\varepsilon_r = 10$, $\sigma = .01$ mhos/m, $r_2 = 10$ m, $\theta_2 = 10°$, and $\phi_2 = 0°$.

$$P(z) = \left[P(z') - 2j \frac{\partial}{\partial x} g(z')\right] \exp\left[jc\,(z-z')\right] + 2j \frac{\partial}{\partial x} g(z)$$

$$-2c \exp(jcz) \int_{z'}^{z} \frac{\partial}{\partial x} g(z) \exp(-jcz)\, dz \quad . \qquad (18)$$

Several important features of the integral appearing in (18) will now be pointed out. First, as seen in Table 1, the results of integration are quite stable as a function of z'. For example, for $z'/\lambda > 1.0$ the difference between the exact Π_z values and those computed by using the expression derived here is less than 5% for $z/\lambda = 0.42$. Second, for the range of parameters investigated thus far, the integrand has been found to be quite smooth. This is due partly to the fact that the singularity associated with g is located at P_2 (see Fig. 1) and because the range of integration is less than one wavelength. As a result of these properties, the numerical integration in (18) can be carried out quite rapidly.

Finally, because the parameter z appears only in the limit of the integral, and not in the integrand itself, the values of vector potential Π_z along an entire vertical line can be rapidly generated by simply marching on the incremental integration steps. This is in contrast to the conventional integrals where the entire integrand must be recomputed for each value of the observation point z.

4. Numerical Results and Conclusions

Tables 2-5 compare the accuracy of the present ${}_0\Pi_x$ and Π_z expressions, given in (11) and (15), with those obtained via the exact and asymptotic evaluations of the Sommerfeld integrals. The present technique is virtually as accurate as the exact integration for a relatively high conducting ground (or $|\kappa|$ large) such as sea water (see Tables 2-3) even for extremely small image to observation point distance r_2/λ. For less conducting grounds (or $|\kappa|$ small), the approximation of γ_2 defined by (9) introduces some errors as r_2/λ decreases (see Tables 4-5). However the results are still useful for most practical antenna problems and remain superior to those derived by using the RCM method.

The procedure has been successfully tested for a wide variety of r_2, θ_z, and $|\kappa| > 5$. On the Cyber-175 computer, the evaluation of a ${}_0\Pi_x$ and Π_z pair for a given observation point required ~ 5 msec for the present technique, while the RCM method needed ~ 1 msec, and a recent efficient Sommerfeld exact integration technique [5,8] typically required 40-60 msec of computing time.

Although not discussed here, it is worthwhile to mention that the problem of a vertical dipole radiating over a lossy ground can be handled in a similar manner. The analysis of various antenna structures, comprising both horizontal and vertical wire sections over a lossy ground, will appear in a future communication.

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